

PUTTING THE CALIFORNIA MATHEMATICS STANDARDS INTO PRACTICE

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GOALS FOR TODAY


- 1. Become (more) familiar with the Standards for Mathematical Practice.**
- 2. See/imagine what it looks like to engage students in practices of reasoning and sense making in the classroom.**
- 3. Develop deeper knowledge of rational number concepts.**

BRAINSTORM



Write down as much as you can to describe the **habits of doing math** the typical student has by the 6th grade.

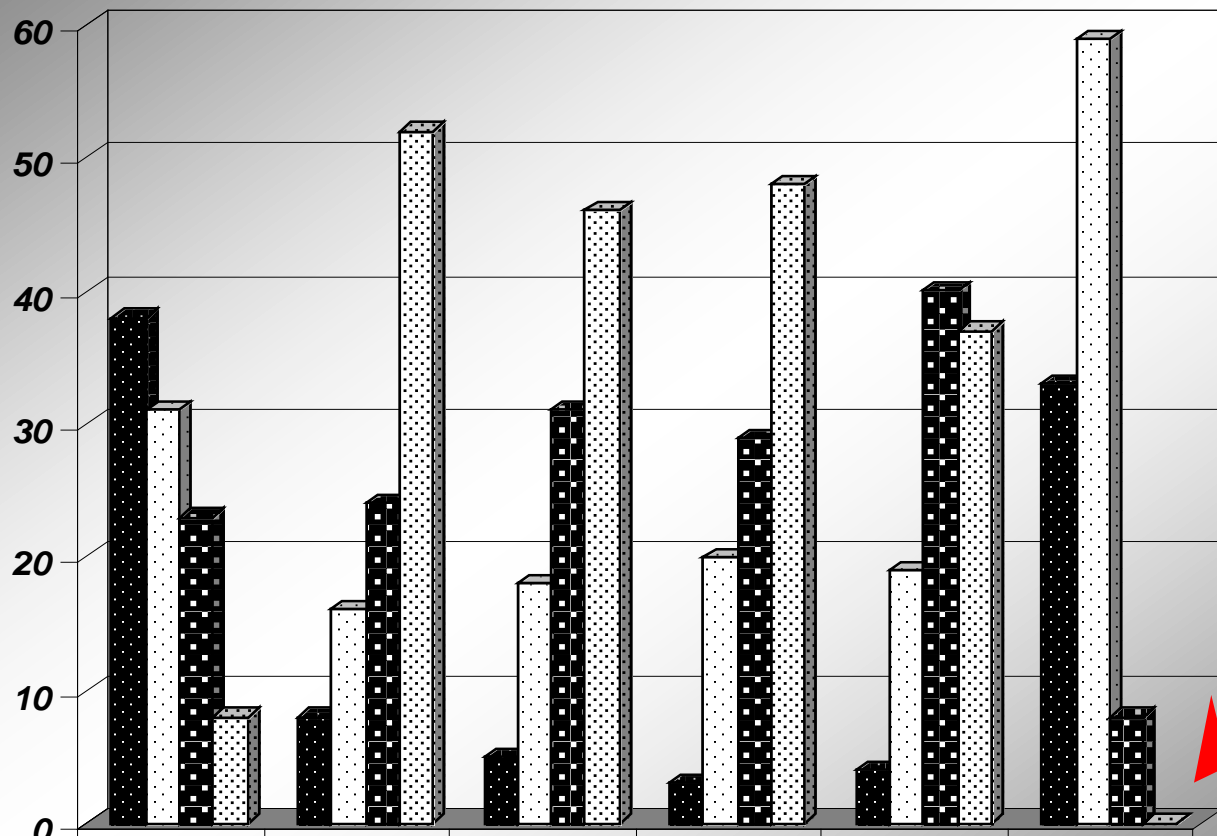
hab·it

/'habit/ 

Noun

A settled or regular tendency or practice, esp. one that is hard to give up.

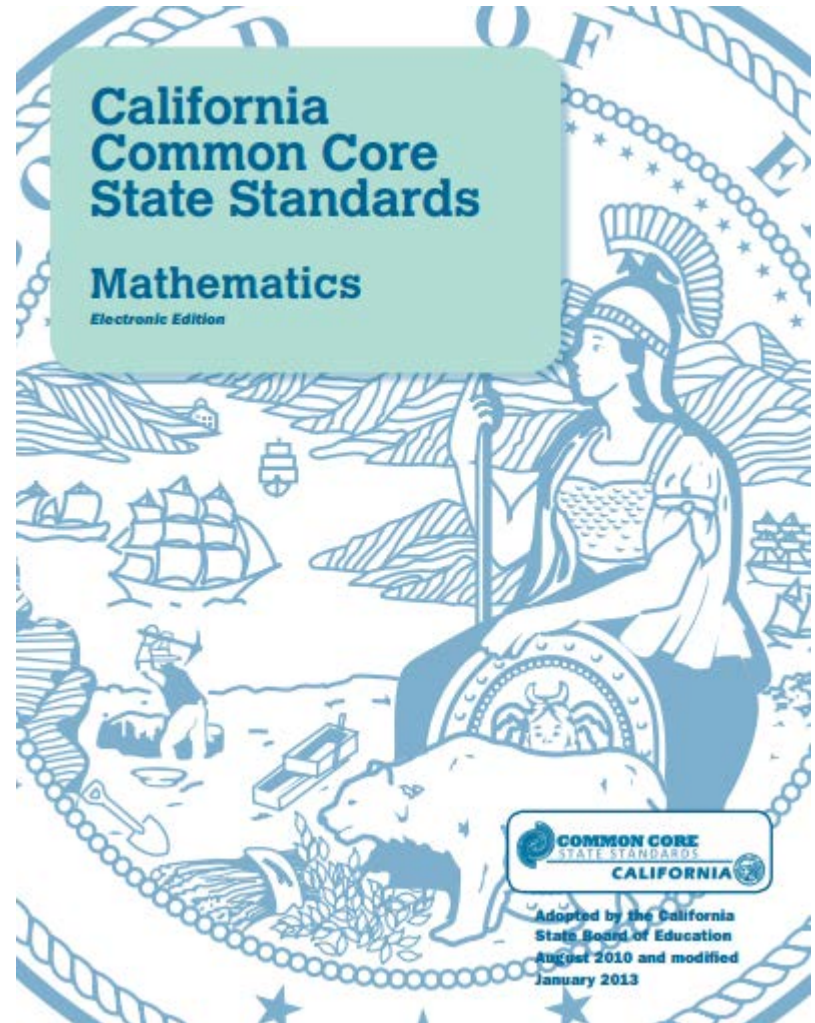
FOCUS OF U.S. MATH LESSONS



■ Giving Results Only	38	8	5	3	4	33
□ Using Procedures	31	16	18	20	19	59
■ Stating Concepts	23	24	31	29	40	8
■ Making Connections	8	52	46	48	37	0

THREE SHIFTS IN MATHEMATICS

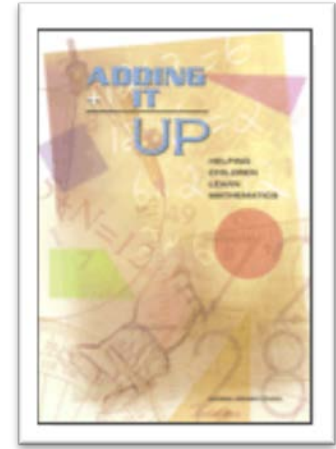
1. **Focus:** Identify key concepts, relationships, and skills that students will learn in depth
2. **Coherence:** Knowledge builds across grades and links to major topics within grades
3. **Rigor:** Require conceptual understanding and procedural fluency; able to apply these to solve real world and non-routine problems



STANDARDS FOR MATHEMATICAL PRACTICE

Mathematically proficient students routinely...

1. ***make sense*** of problems and ***persevere*** in solving them;
2. ***reason*** abstractly and quantitatively;
3. ***construct*** viable arguments and ***critique*** the reasoning of others;
4. ***model*** with mathematics;
5. ***use*** appropriate tools strategically;
6. ***attend to*** precision;
7. ***look for*** and make use of structure;
8. Look for and ***express*** regularity in repeated reasoning.



 Principles & Standards
for SCHOOL MATHEMATICS
HIGHER STANDARDS for OUR STUDENTS
...HIGHER STANDARDS for OURSELVES

ACTIVITY: RATIONAL AND IRRATIONAL NUMBERS

- 1. Using two sticky notes, write one rational and one irrational number.**
- 2. Discuss with a partner why you selected the numbers and how you know whether they are rational or irrational.**
- 3. Stick your numbers in either the Rational or Irrational category on the board.**

WHAT ARE U.S. STUDENTS LEARNING?

Institute of Education Studies report (2010)

- 50% of 8th-graders could not order three fractions from least to greatest.
- Fewer than 30% of 17-year-olds correctly identified 0.029 as equivalent to $\frac{29}{1000}$.

Why? Moving from whole numbers to rational numbers is DIFFICULT.

**Students need opportunities to make sense of rational numbers, conceptually and procedurally...and in relation to whole number concepts.*

EXAMINING STUDENT WORK

- What does this tell us about the student's understanding?
- What are some possible ways to address the misunderstanding?

The sum of $\frac{1}{12}$ and $\frac{7}{8}$ is closest to

- A. 20
- B. 8
- C. $\frac{1}{2}$
- D. 1

Explain your answer.

$$\frac{1}{12} + \frac{7}{8} = \frac{2}{24} + \frac{21}{24} = \frac{23}{24} \text{ is closest to } 20.$$

ACTIVITY: ORDERING FRACTIONS

Without using decimal notation or “tricks”
order these fractions from smallest to largest. Provide evidence of how you determined the correct order.

$$\frac{2}{5}, \frac{7}{8}, \frac{1}{3}, \frac{9}{10}, \frac{12}{7}, \frac{13}{11}, \frac{7}{9}, \frac{5}{8}, \frac{5}{6}$$

STRATEGIES FOR COMPARING FRACTIONS

For each strategy, explain what it means and give 1-2 examples. Share your work with a partner to get feedback.

1. Same size pieces
2. Same number of pieces
3. Distance from a whole
4. Benchmark

Link to Standards

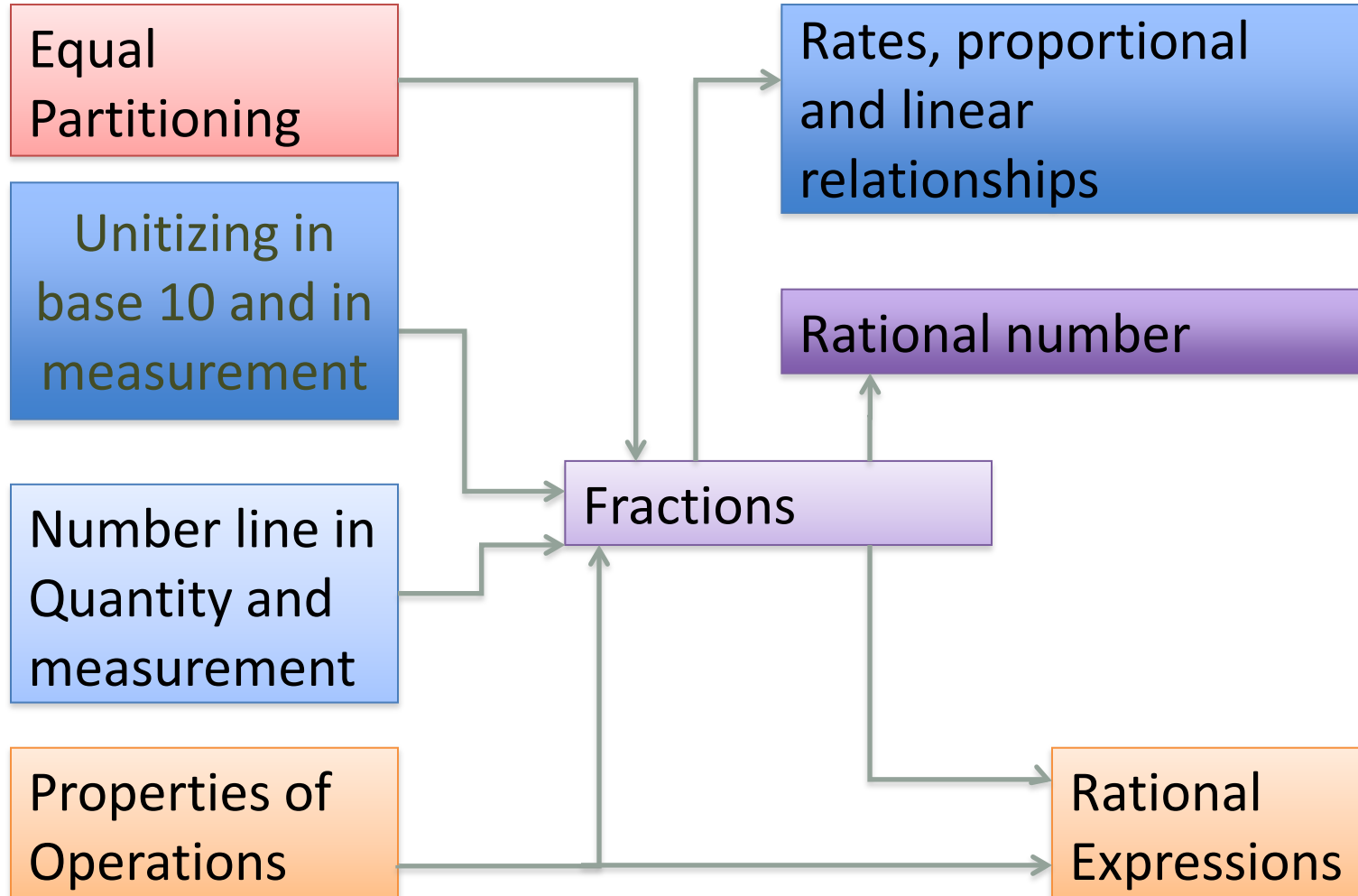
- Gr. 3: Develop an understanding of fractions as numbers
- Gr. 4: Extend understanding of fraction equivalence and ordering

Basis for Fraction Learning

K - 2

3 - 6

7 - 12



UNITIZING

It is important that students see a group(s) of objects or abstractions like “tens” as a unit(s) that can be counted.

- *Whatever can be counted can be added.*
- **Knowledge of whole number arithmetic can be extended to newly “unitized” objects, like counting tens in base 10, adding standard lengths such as inches, and adding unit fractions such as $\frac{1}{4}$.**

LEARNING TRAJECTORY

Equal partitioning and *unitizing* are concepts that must be built up from whole number operations through to algebra.

Table 1. Development of Equal Partitioning and Unitizing

Objects	$3 \text{ objects} + 5 \text{ objects} = 8 \text{ objects}$
Pure numbers	$3 \text{ ones} + 5 \text{ ones} = 8 \text{ ones}$
Groups of objects	$3 \text{ groups of } 10 \text{ objects} + 5 \text{ groups of } 10 \text{ objects} = 8 \text{ groups of } 10 \text{ objects} = 80$
Groups of 10 ones are tens	$3 \text{ tens} + 5 \text{ tens} = 8 \text{ tens}$
Equal lengths are units	$3 \text{ inches} + 5 \text{ inches} = 8 \text{ inches}$
A length can be equipartitioned into equal sized units.	$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$
A part of 1 inch, $\frac{1}{4}$ inch, can be counted, added, etc. as a unit	$3 (\frac{1}{4} \text{ inches}) + 5 (\frac{1}{4} \text{ inches}) = 8 (\frac{1}{4} \text{ inches})$
Unit fractions as pure numbers can be counted, added and multiplied	$3(\frac{1}{4}) + 5(\frac{1}{4}) = 8(\frac{1}{4}) = 8/4$
Expressions with letters can be read as uncalculated numbers	$3(x + 1) + 5(x+1) = 8(x+1)$

REASONING ABOUT FRACTION DIVISION

1. What question might this expression represent?
2. Work this out in a way that *makes sense*.

$$1\frac{1}{2} \div \frac{1}{3}$$

Share your reasoning with your neighbor(s). What justifies your method?

BIG IDEAS OF FRACTION (GRADES 3-6)

Gr. 3: Develop an understanding of fractions as numbers

Gr. 4: Extend understanding of fraction equivalence and ordering

Gr. 4: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers

Gr. 4: Understand decimal notation for fractions, and compare decimal fractions

Gr. 5: Use equivalent fractions as a strategy to add and subtract fractions

Gr. 5: Apply and extend previous understandings of multiplication and division to multiply and divide fractions

Gr. 6: Apply and extend previous understandings of multiplication and division to divide fractions by fractions

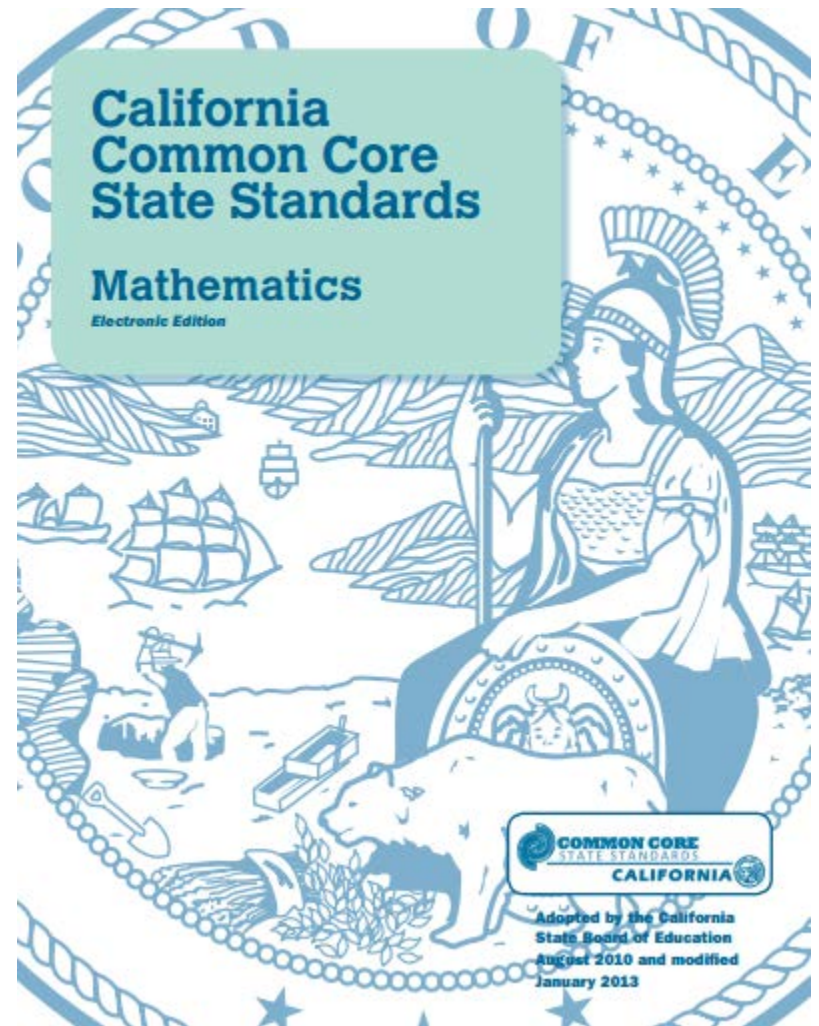
STANDARDS FOR MATHEMATICAL PRACTICE

Which of these were you engaged in today?

1. **make sense** of problems and **persevere** in solving them;
2. **reason** abstractly and quantitatively;
3. **construct** viable **arguments** and **critique** the **reasoning** of others;
4. **model** with mathematics;
5. **use** appropriate tools strategically;
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REMINDER: THREE SHIFTS IN MATHEMATICS

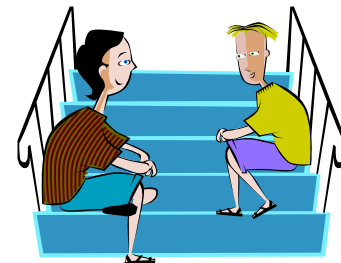
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Envisioning the Standards for Mathematical Practice – Pedagogical Considerations

With 1-2 partners, discuss:

- *What sort of activities must happen in classrooms if students are to attain proficiency with these practices?*
- *What would a teacher need to do to support students in developing these habits?*



PRINCIPLES TO ACTIONS: ENSURING MATHEMATICAL SUCCESS FOR ALL



Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

CALIFORNIA MATHEMATICS STANDARDS RESOURCES

General

- California Dept Educ <http://www.cde.ca.gov/re/cc/>
- OCDE Common Core <http://www.ocde.us/commoncoreca/>

Mathematics Teaching and Planning

- NCTM's *Principles to Actions* <http://www.nctm.org/PrinciplestoActions/>
- NCTM's *Making it Happen* e-book and resources:
<http://www.nctm.org/mih>
- Content Progressions: <http://ime.math.arizona.edu/progressions/>
- Common Core Toolbox: <http://www.ccsstoolbox.com>
- Illustrative Mathematics: www.illustrativemathematics.org
- National Science Digital Library <http://nsdl.org/commcore/math>
- Share My Lesson
<http://www.sharemylesson.com/article.aspx?storyCode=50000148>

Assessment

- Mathematics Assessment Project
<http://map.mathshell.org/materials/tasks.php>
- Smarter Balanced Assessment Consortium
<http://www.smarterbalanced.org/>