Progressing with Fraction Understanding in the CCSS

Mark Ellis, Ph.D., NBCT
@ellismathed
Secondary Education
CSU Fullerton

Jenny Bae, M.A., NBCT
Middle Grades Math Coach
Norwalk La Mirada USD

Goals for this Session

1. Develop deeper knowledge of rational number concepts and operations.
2. Become (more) familiar with the Standards for Mathematical Practice.
3. See/imagine what it looks like to engage students in practices of reasoning and sense making in the classroom.

New Standards for Mathematics

- Understand the concepts behind the calculations
- Skill in reasoning and communicating mathematically
- Flexibility to solve non-routine problems

Non-Negotiable Positions

- Math makes sense. (*And if it does not, ASK!*)
- Mathematics learning should be coherent.
- Students learn best through active engagement.
  - *thinking, reasoning, connecting, communicating, critiquing, revising, and extending knowledge*

Guiding Question for Teachers

How does my instruction support all students in developing mathematical authority?

What are U.S. Students Learning?

Institute of Education Studies report (2010)

- 50% of 8th-graders could not order three fractions from least to greatest.
- Fewer than 30% of 17-year-olds correctly identified **0.029** as equivalent to $\frac{\mathbf{29}}{\mathbf{1000}}$.
- Even students who can "follow rules" lack understanding and flexibility.

Why? Moving from whole numbers to rational numbers is DIFFICULT.

*Students need opportunities to make sense of rational numbers, conceptually and procedurally…and in relation to whole number concepts.

Examining Student Work

The sum of $\frac{1}{12}$ and $\frac{7}{8}$ is closest to

A. 20
B. 8
C. ½
D. 1

Explain your answer.

$\frac{1}{12} + \frac{7}{8} = \frac{2}{24} + \frac{21}{24} = \frac{23}{24}$ is closest to 20.

- What does this tell us about the student's understanding?
- What does this tell us about the student's misconception?

Fraction Mantras

- Fractions are numbers.
- Fractions operations are logical, not magical.

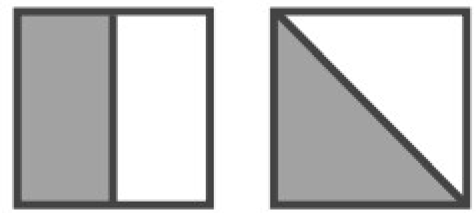
To (re)visit fraction concepts, see:
www.illustrativemathematics.org/progressions

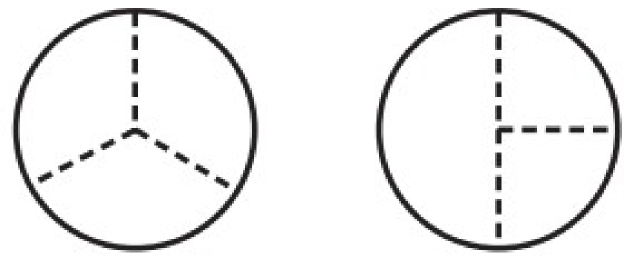
Key to Fraction Success—Unitizing

- Build on concept of "unit" from whole numbers to make sense of fractions as numbers.
- Use same representations as whole number operations so fractions are seen as part of a continuum.
- Contextualize work with fractions to give meaning to operations.

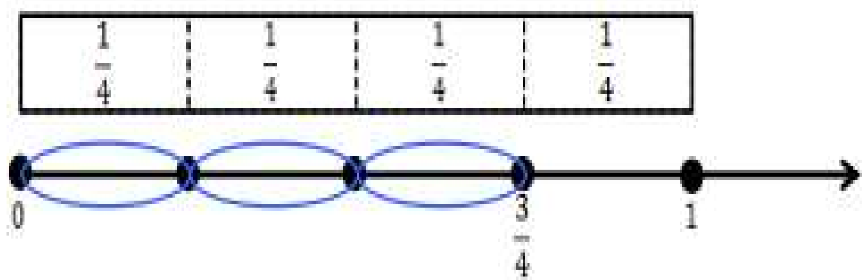
Fraction Representations

- Grades 1-2: halves, fourths, and thirds as terms related to geometric shapes
- Grade 3: fractions on number line and with visual models
- Grades 4-6: fraction operations on number line and with visual models to support sense-making and connection to procedures

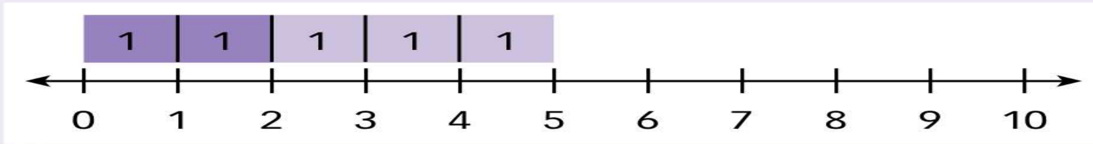




thirds not thirds



Connect Fraction and Whole Number Addition

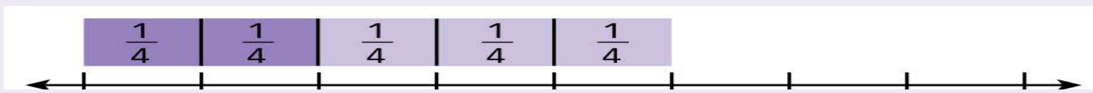


When you add 2 + 3, you are putting ones together.

Think

Adding fractions means joining or putting together parts of the same whole.

You can put a segment with a length of $\frac{2}{4}$ and a segment with a length of $\frac{3}{4}$ next to each other to show $\frac{2}{4} + \frac{3}{4}$.



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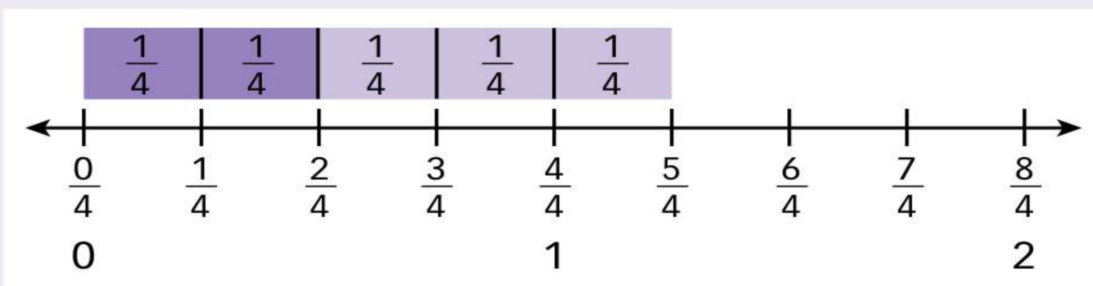
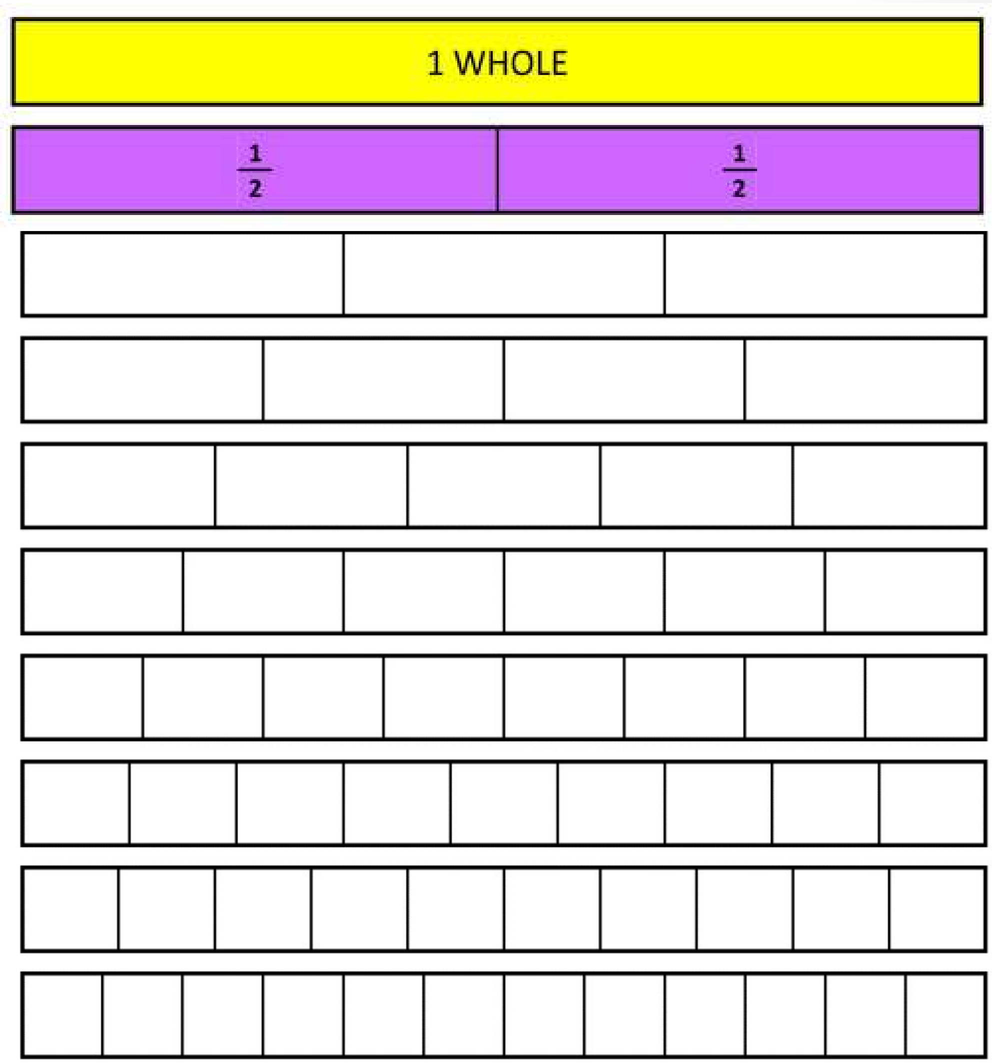


When you add $\frac{2}{4} + \frac{3}{4}$, you are putting one-fourths together.

Page from [*Ready Math* grade 4 workbook].

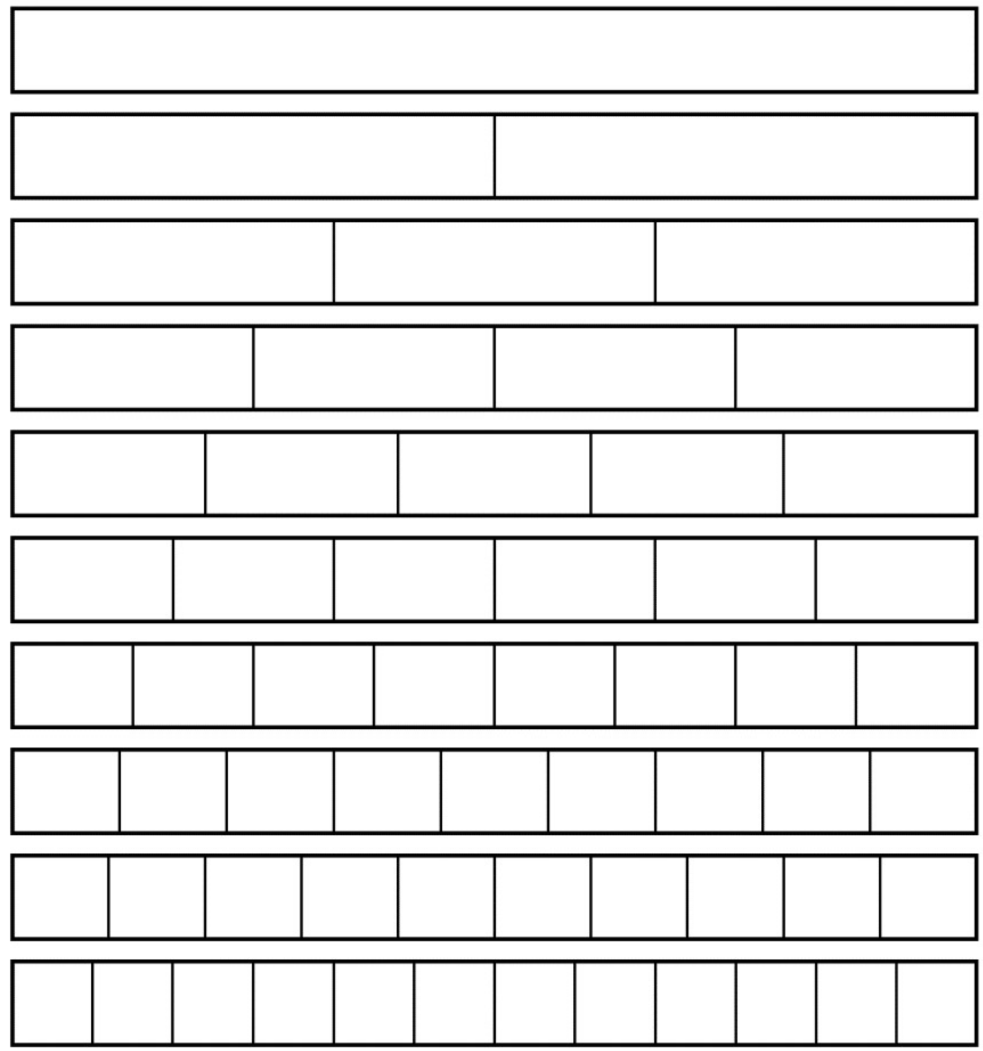
Try This: Fraction Strips

- Label your fraction strips based on this example.
- Choose 2 fractions with unlike denominators and use the strips to compare them. Repeat this for another pair.
- Check with a partner.



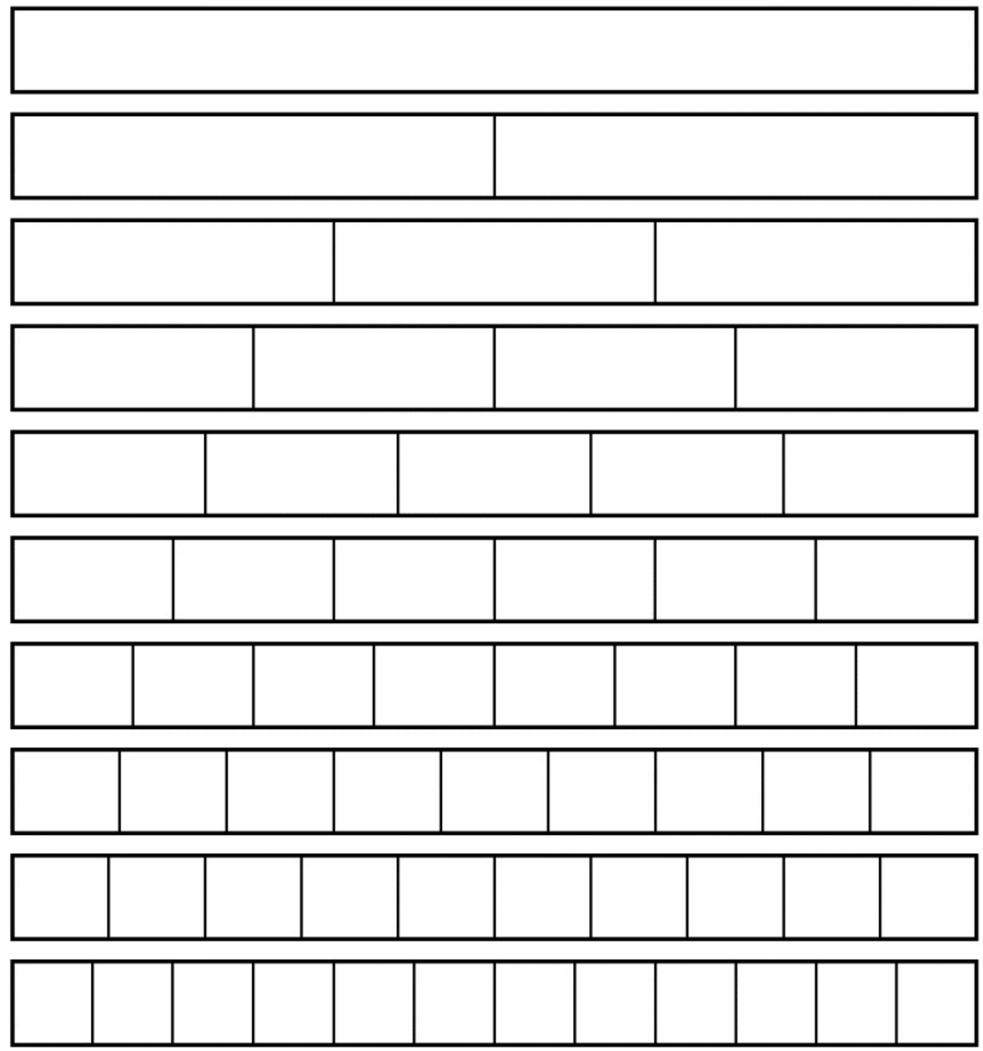
Understanding Fraction Addition

- How might we figure out $\frac{1}{2} + \frac{1}{4}$?
- Come up with more than one strategy.



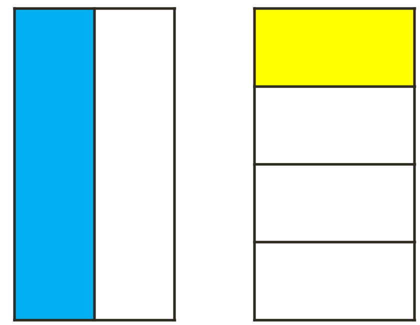
Understanding Fraction Addition

- How might we figure out $\frac{1}{2} + \frac{2}{3}$?
- Come up with more than one strategy.

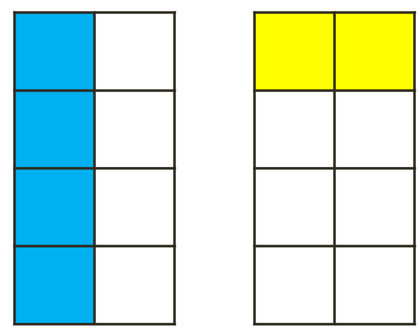


Addition Using Array Models

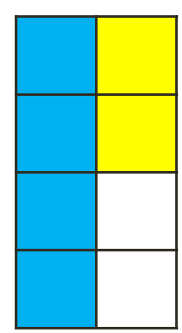
$$\frac{1}{2} + \frac{1}{4}$$



$$\frac{4}{8} + \frac{2}{8}$$



$$\frac{6}{8}$$

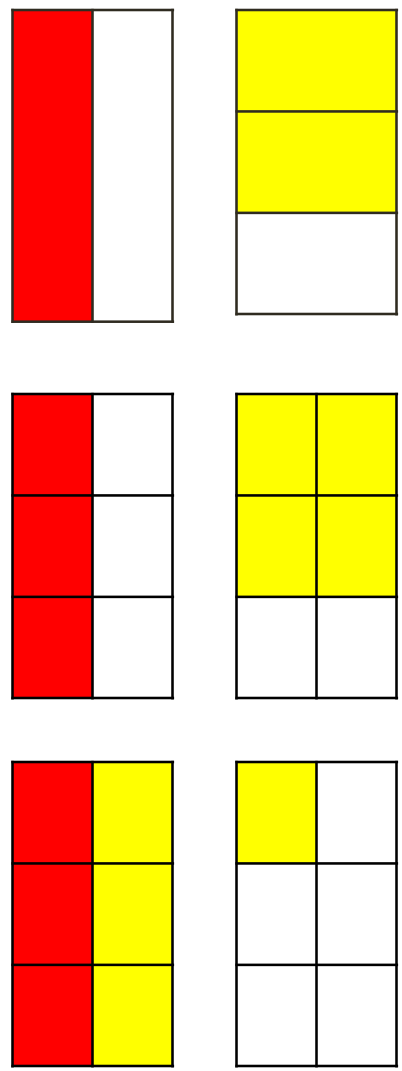


Addition Using Array Models

$$\frac{1}{2} + \frac{2}{3}$$

$$\frac{3}{6} + \frac{4}{6}$$

$$\frac{7}{6} \; or \; 1\frac{1}{6}$$



Use Any Model/Method that You Can Justify Mathematically

$$\frac{3}{4}+\frac{1}{3}$$

$$\frac{3}{8}+\frac{1}{4}$$

$$\frac{11}{12}+\frac{9}{10}$$

Describe and Justify an Algorithm for Adding Two Fractions

al·go·rithm

noun

1. a process or set of rules to be followed in calculations or other problem-solving operations, especially by a computer.

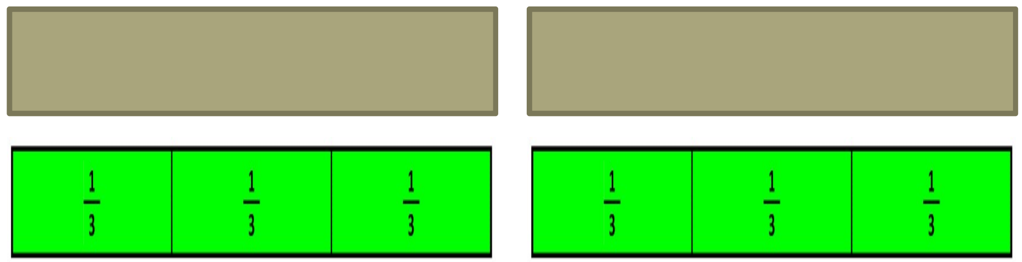
Reasoning about Fraction Division

1. What question might this expression represent?
2. Represent this in a way that *makes sense mathematically*.

$$2 \div \frac{1}{3}$$

- *What justifies your method?*

Measurement Model: How Many 1/3s are in 2?



$$2 \div \frac{1}{3} = 6$$

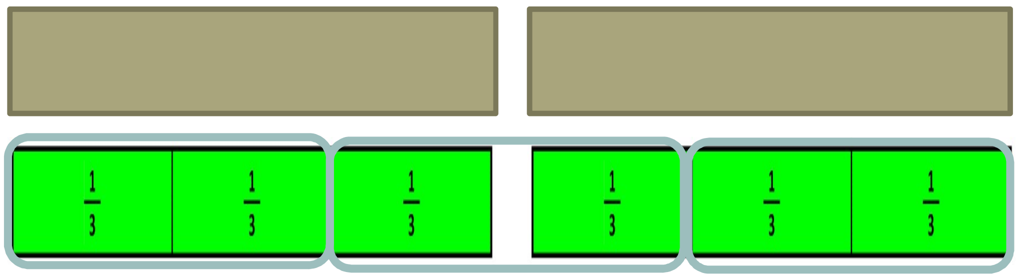
Reality Check: What does the "6" represent? Does this make sense?

Reasoning about Fraction Division

1. Represent this in a way that *makes sense*.
2. *What justifies your method?*
3. How is this related mathematically to the previous problem?

$$2 \div \frac{2}{3}$$

Measurement Model: How Many 2/3s are in 2?



$$2 \div \frac{2}{3} = 3$$

Reality Check: What does the "3" represent? Does this make sense?

Make a Visual Model, Then Calculate Based on the Model

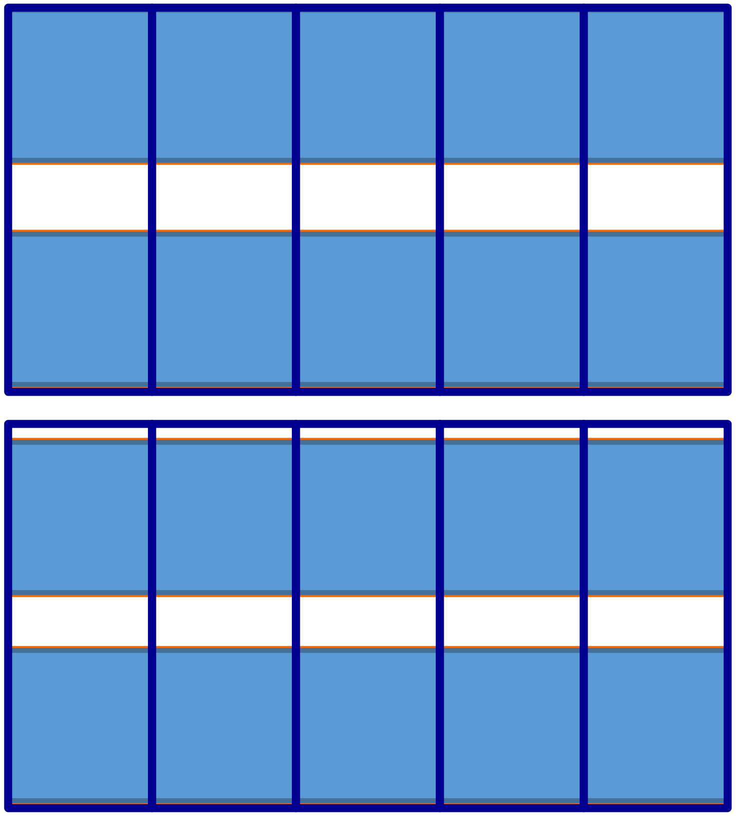
$$4 \div \frac{1}{5} \qquad 4 \div \frac{2}{5}$$

$$5 \div \frac{1}{4} \qquad 5 \div \frac{3}{4}$$

$$6 \div \frac{1}{8} \qquad 6 \div \frac{5}{8}$$

What do you notice through repeated reasoning about division by a fraction?

What Did You Notice through Repeated Reasoning?



$$4 \div \frac{1}{5} \Rightarrow 4 \div \frac{2}{5}$$

$$20 \Rightarrow 10$$

What is a rule you can create?

What justifies your rule?

Does it always work?

“Just Invert and Multiply” – *WHY?*

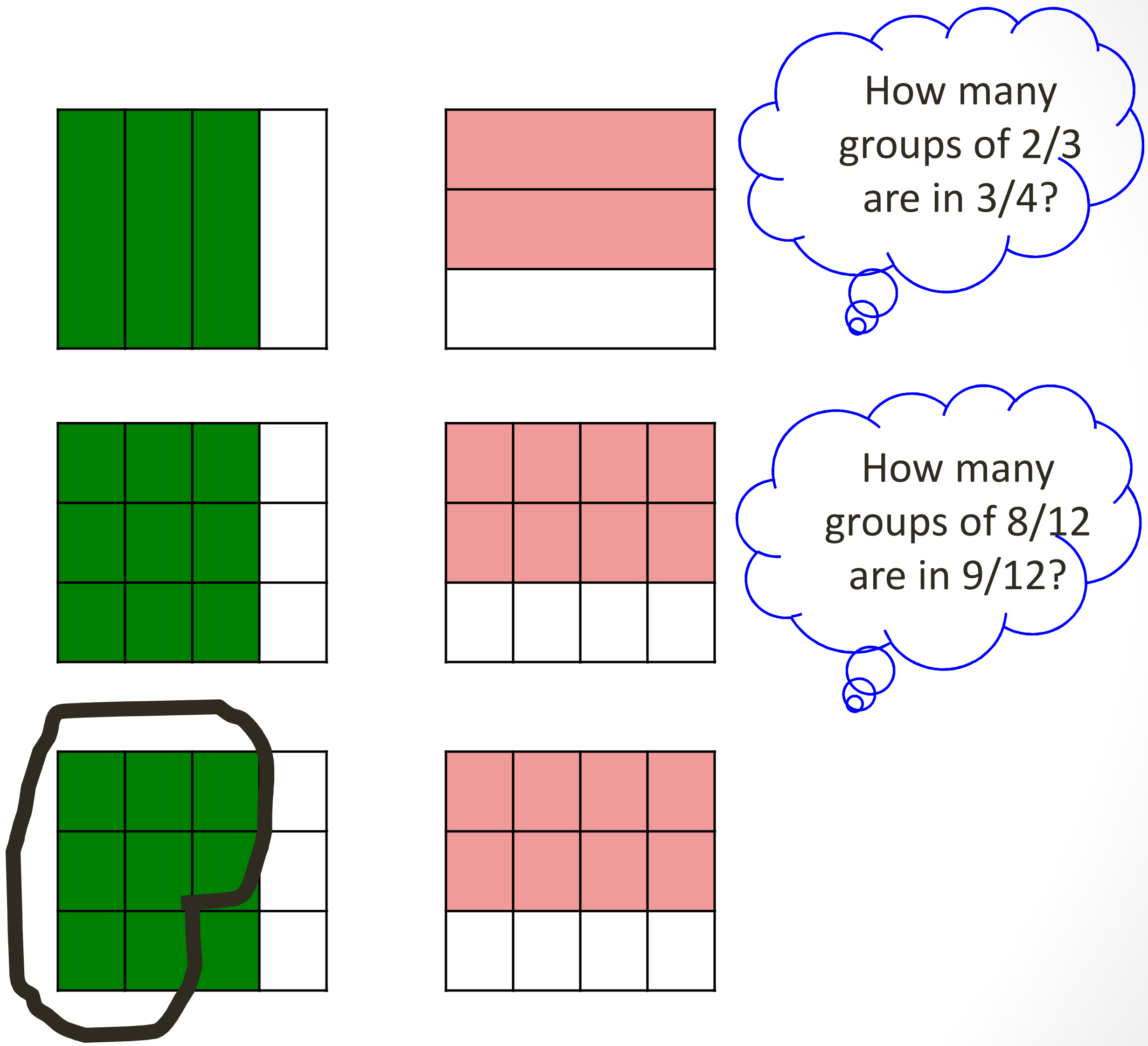
- Talk with elbow partners about a mathematical justification of this commonly followed rule for dividing by a fraction.

Division using Array Models

$$\frac{3}{4} \div \frac{2}{3}$$

$$\frac{9}{12} \div \frac{8}{12}$$

$$1\frac{1}{8}$$

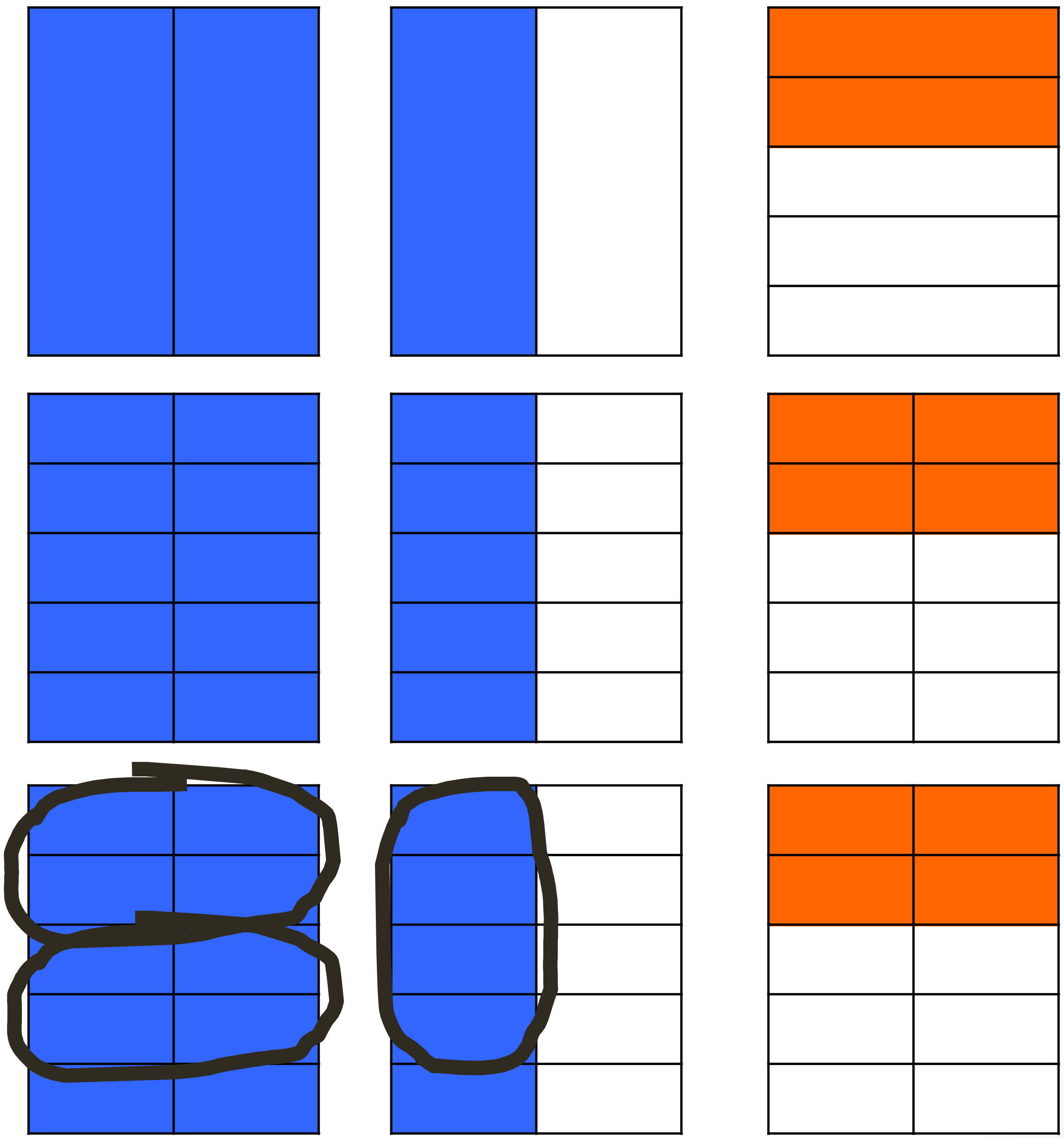


Division using Array Models

1 ½ ÷ 2/5

15/10 ÷ 4/10

3 and 3/4



How many groups of 2/5 are in 1 ½ ?

Try This! Garden Problem

Maren ordered 4 bags of soil for her raised flower gardens. Each garden needs 3/4 of a bag of soil.

- How many gardens can she fill completely with soil?
- How much soil does she have left?

Show your work with both visual and symbolic representations.

Debrief

- Why is there a discrepancy between the answer from the picture and the answer from the algorithm?
- In what context would the traditional algorithm apply?
- How did using a visual model help you make sense of what the problem was asking?

Standards for Mathematical Practice

Which of these were you engaged in?

1. make sense of problems and persevere in solving them;
2. reason abstractly and quantitatively;
3. construct viable arguments and critique the reasoning of others;
4. model with mathematics;
5. use appropriate tools strategically;
6. attend to precision;
7. look for and make use of structure;
8. Look for and express regularity in repeated reasoning.

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

New Standards for Mathematics

- Understand the concepts behind the calculations
- Skill in reasoning and communicating mathematically
- Flexibility to solve non-routine problems

References

- Baroody, A., Bajwa, N. & Eiland, M. (2009). Why can't Johnny remember the basic facts? *Developmental Disabilities Research Reviews ,15*(1), 69–79.
- Ellis, M. (2015, March 4). Mastering the most challenging math standards with rigorous instruction. *Education Week* webinar.
- Fuson, K., Kalchman, M., & Bransford, J. (2005). Mathematical understanding: An introduction. In Donovan and Bransford (Eds.), *How Students Learn: Mathematics in the Classroom* (pp. 217-256). National Research Council, Washington, D.C.: National Academies Press
- Institute of Education Sciences (2010). Developing effective fractions instruction for grades K-8. http://ies.ed.gov/ncee/wwc/practiceguide.aspx?sid=15
- National Council of Teachers of Mathematics (2014). *Principles to Actions: Ensuring Mathematics Success for All*. NCTM. Reston, VA.