Envisioning a New Normal for Middle and High School Mathematics

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One-Minute Brainstorm

- Write down the habits of doing math the typical student has by the 10th grade.

hab·it

/ˈhabit/

Noun

A settled or regular tendency or practice, esp. one that is hard to give up.

Reflection

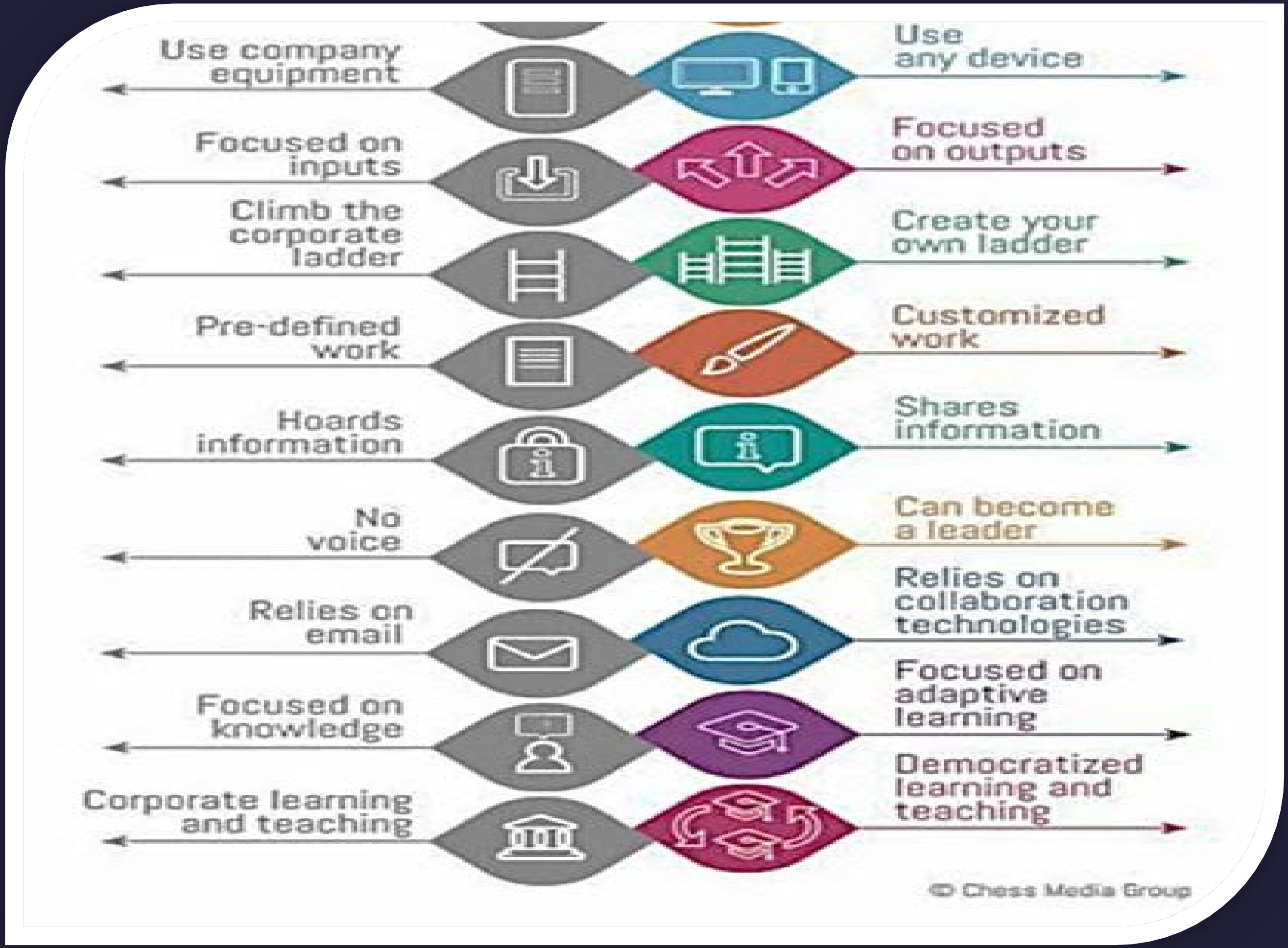
Reflection

- Where did students learn these habits of doing math?
- Whose responsibility is it to change these if they are not productive habits?

Music Then...

Music Now...

Workplace Then and Now



Beliefs about Learning Mathematics

"Then"	"Now"
Students possess different innate levels of ability in mathematics, and these cannot be changed by instruction. Certain groups or individuals have it while others do not.	Mathematics ability is a function of opportunity, experience, and effort—not of innate intelligence. Mathematics teaching and learning cultivate mathematics abilities. All students are capable of participating and achieving in mathematics, and all deserve support to achieve at the highest levels.

Principles to Actions, p. 63

Math Classrooms (then…)

Majority of lessons **lacked**

- Intellectual Engagement
- Productive Questioning
- Focus on Sense-Making

About one-third of lessons had evidence of

- Low Respect
- Low Rigor
- Lack of Access for Some Students

"In a 9th grade teacher's efforts to help his students better understand how to solve equations and inequalities, he asked them to remember and repeat the procedures he had demonstrated. The teacher's presentation included prompts such as, 'There's the variable, what's the opposite?' and 'Tell me the steps to do.' He did very little to engage students with the content; two students slept through the teacher's entire presentation, and one read a magazine."

http://www.horizon-research.com/horizonresearchwp/wp-content/uploads/2013/04/highlights.pdf

Math Classrooms Now

Content standards promote coherence and rigor, a balance of conceptual and procedural knowledge, and a new set of habits.

Mathematically proficient students routinely…

1. *make sense* of problems and *persevere* in solving them;
2. *reason* abstractly and quantitatively;
3. *construct* viable arguments and *critique* the reasoning of others;
4. *model* with mathematics;
5. *use* appropriate tools strategically;
6. *attend to* precision;
7. *look for* and make use of structure;
8. Look for and *express* regularity in repeated reasoning.

CA CCSS Math Objectives

7.NS.2.d

Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

8.NS.1

Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Base 10 Place Value Number System

100,000	10,000	1,000	100	10	1	0.1	0.01	0.001	0.0001
100,000	10,000	1,000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$	$\frac{1}{10,000}$
10^5	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	Ten-thousandths

Read this number: 3.14

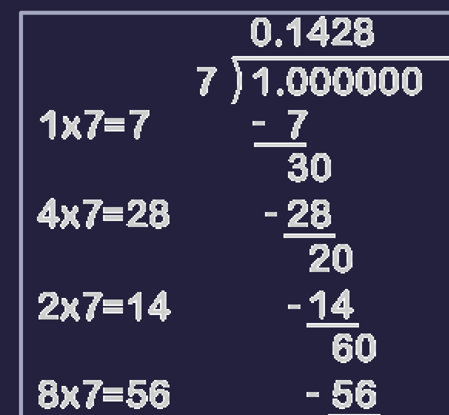
- What does each digit represent?

ACTIVITY: Terminating or Repeating?

- When you look at a common fraction, can you tell right away whether it will be represented with a terminating decimal or a repeating decimal?
- Maybe you already know some examples.
  - Common fractions that you believe have *terminating* decimal representations?
  - Common fractions that you believe have *repeating* decimal representations?
- **Let's generate some data to "play" with!**

Use Any Means You Like to Find These…*But Be Sure Your Results are Accurate*

Fraction	Expansion
$\frac{1}{2}$	
$\frac{1}{3}$	
$\frac{1}{4}$	
$\frac{1}{5}$	
$\frac{1}{6}$	
$\frac{1}{7}$	$0.\overline{142857}$
$\frac{1}{8}$	0.125
$\frac{1}{9}$	
$\frac{1}{10}$	
$\frac{1}{11}$	
$\frac{1}{12}$	
$\frac{1}{13}$	
$\frac{1}{14}$	
$\frac{1}{15}$	
$\frac{1}{16}$	



Forming & Testing Conjectures

1. Which of the unit fractions from $\frac{1}{2}$ to $\frac{1}{16}$ have terminating decimals? Circle them.
   – **What *else* do these have in common? Discuss!**
2. Conjecture whether these three fractions, $\frac{1}{18}, \frac{1}{20}, and\ \frac{3}{25}$ will terminate. Turn them into decimals to check whether you were correct. Think about why each did or did not terminate.
3. Try to write a rule about when a unit fraction will terminate.

What’s Mathematically Special?

1/2	1/10
1/4	1/16
1/5	1/20
1/8	1/25

Explanation, Extension, Connection

4. <u>Explanation</u>: Why do some fractions terminate as decimals while others repeat? What explains this mathematically?
5. <u>Extension</u>: Given any fraction that will terminate in decimal form, what determines how many decimal places it will have?
6. <u>Connection</u>: When multiplying decimals, why do we “add” decimal places of the factors to place the decimal point in the product?

Share your ideas on this Padlet

http://padlet.com/ellismathed/fractiondecimal

Debrief

- What are some of the "practices" of mathematics you were engaged in during this activity?
- What aspects of this activity supported your engagement in mathematical reasoning and sense-making?
- What does this sort of learning mean for teachers of mathematics and how we do our work??

Mathematical Practices

1. **make sense of problems and persevere in solving them;**
2. **reason abstractly and quantitatively;**
3. **construct viable arguments and critique the reasoning of others;**
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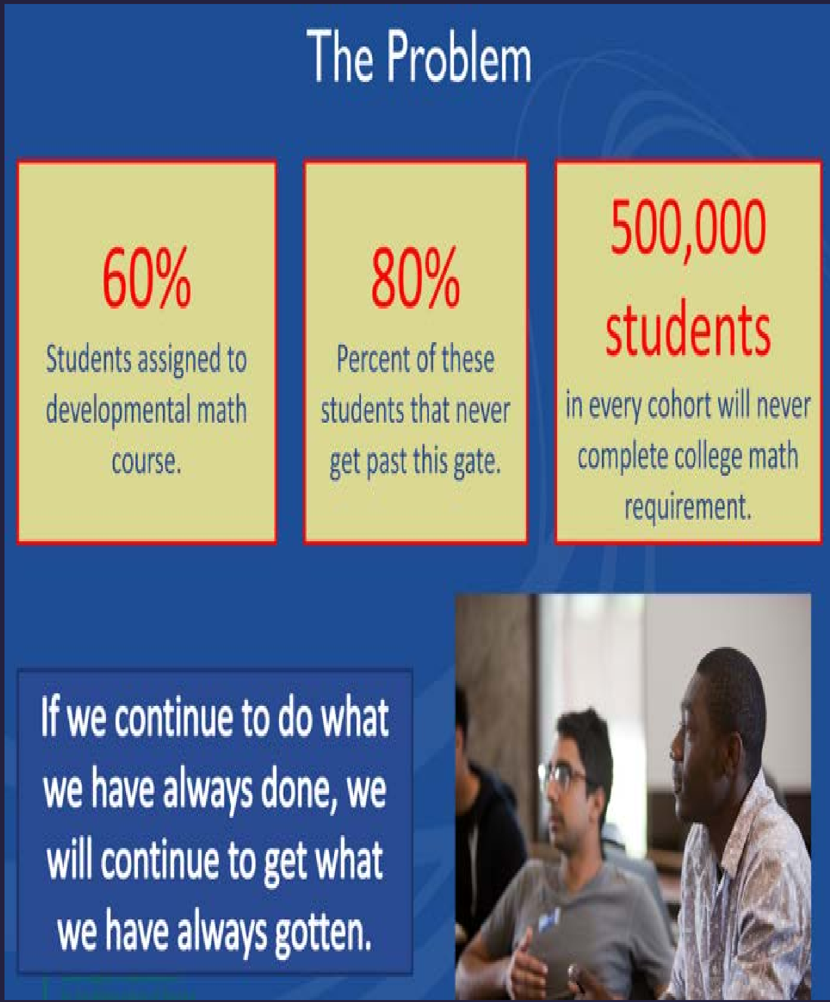
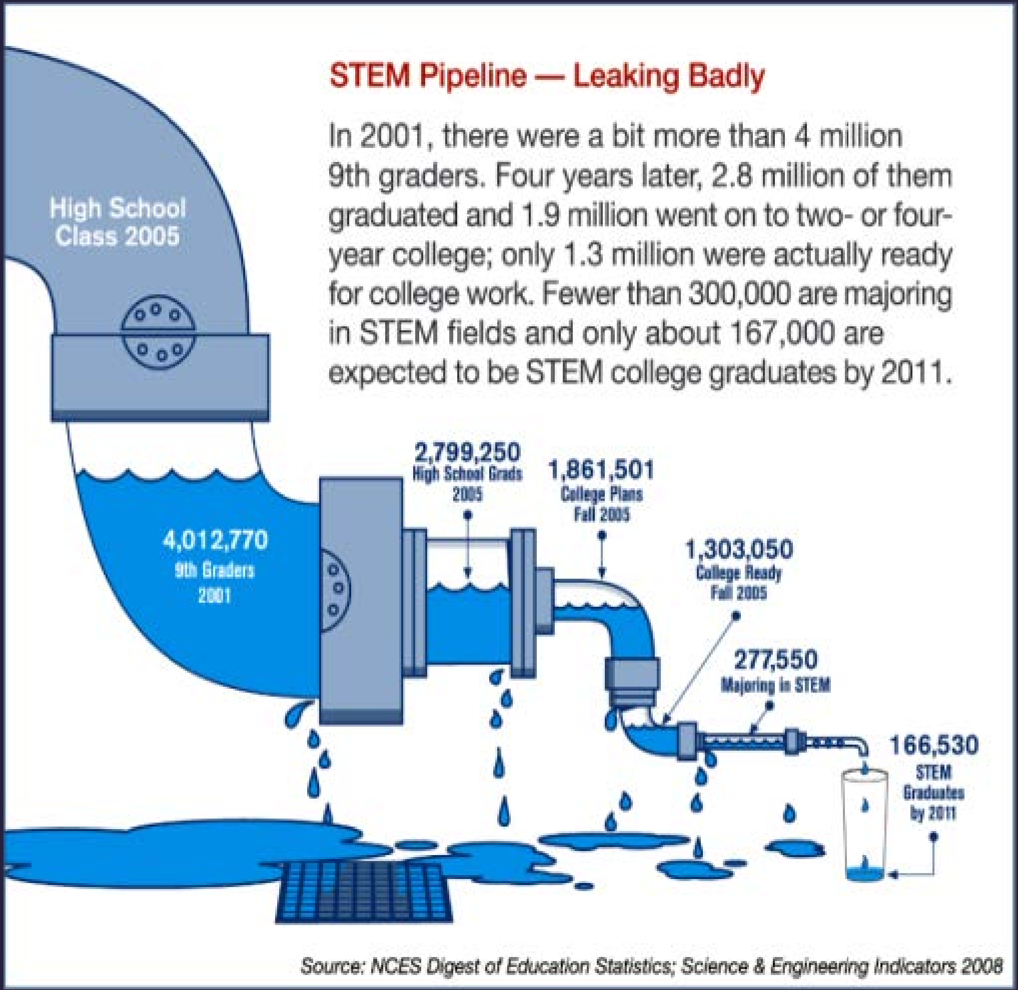
7. **look for and make use of structure;**
8. **Look for and express regularity in repeated reasoning.**

New Practices for Teachers

- **Strategy 1:** Set Norms and Pose Tasks that Elicit Student Reasoning and Promote Sense-Making
- **Strategy 2:** Provide Information, Guidance & Feedback that Values Specific Student-led Processes and Invites Further Learning
- **Strategy 3:** Recognize and Reduce/Remove Barriers to Learning Mathematics

Does This Really Matter to University Faculty?





http://www.calstate.edu/app/GEAC/documents/2015/sept-2015/06-Statway-presentation.pdf

http://wswheboces.org/m/content.cfm?subpage=15

University Mathematics Faculty View

From their high school mathematics courses students should have gained certain approaches, attitudes, and perspectives:

- Perceive mathematics as a way of understanding, not as a sequence of algorithms to be memorized
- Expect that mathematics makes sense
- Are willing to work on mathematical problems requiring time and thought

ICAS Math Doc

University Mathematics Faculty View

From their high school mathematics courses students should have gained certain approaches, attitudes, and perspectives:

- Discuss and write coherently to communicate about mathematical ideas and understanding
- Use computational devices to explore, formulate, and investigate mathematical conjectures
- Use mathematical knowledge to solve unfamiliar problems

ICAS Math Doc

Criteria for “a-g” Math Courses

- Do the assignments expect students to work on problems requiring time and thought that are not solved by merely mimicking examples that have already been seen?
- Does instruction model mathematical thinking where justification is based upon persuasive arguments?
- Do the assessments require that students communicate their reasoning?

University of Minnesota Intro to Biology

http://www.classroom.umn.edu/projects/alc.html

Active Learning

"Active learning engages students in the process of learning through activities and/or discussion in class as opposed to passively listening to an expert. It emphasizes higher-order thinking and often involves group work."

[Scott Freeman et al. PNAS 2014;111:8410-8415]

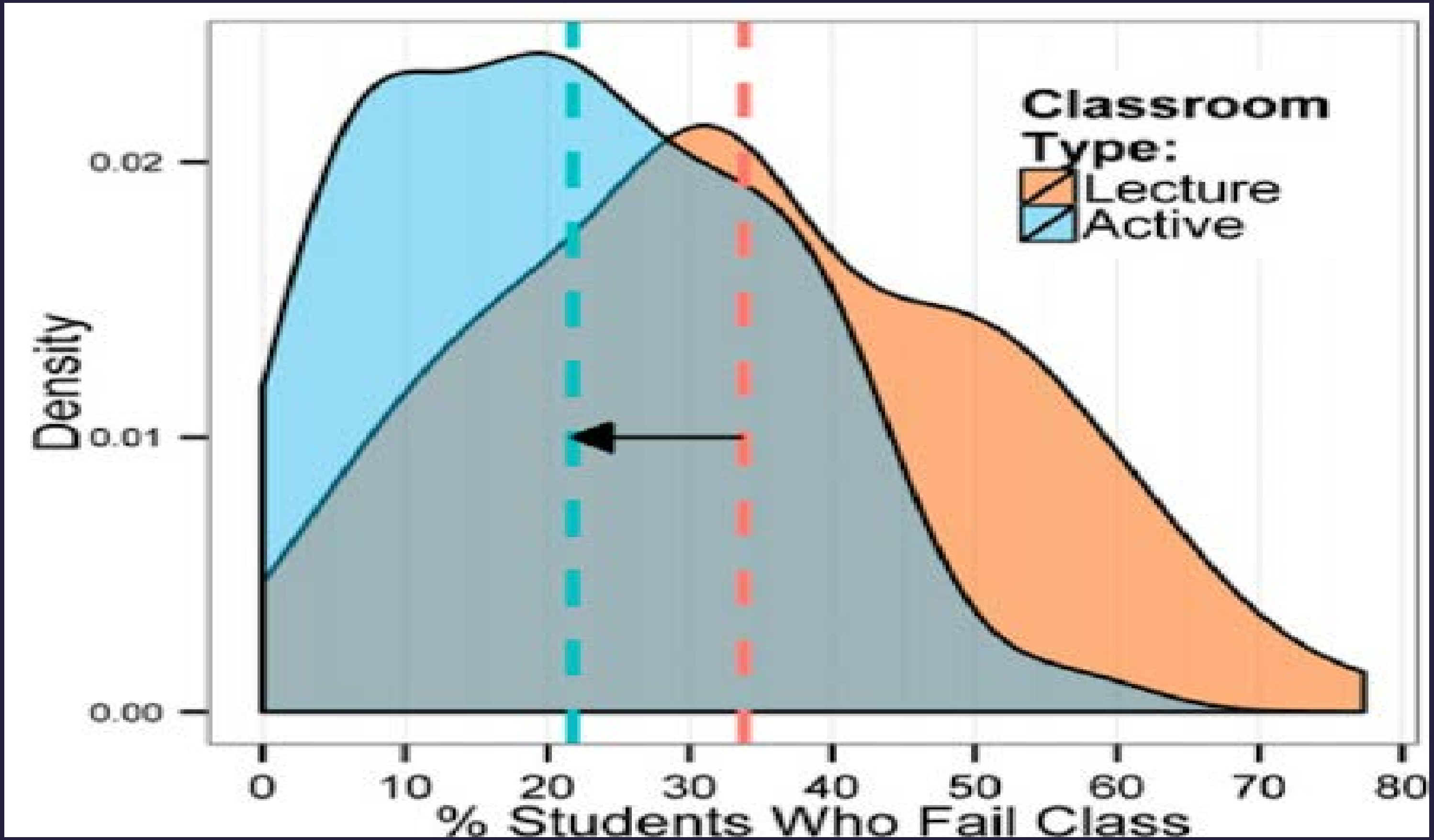
Not about sidelining the teacher…but about supporting meaningful learning!

https://www.youtube.com/watch?v=wont2v_LZ1E and https://www.youtube.com/watch?v=hbBz9J-xVxE

Practices of Active Learning

- **Prepare clear learning outcomes**
- **Probe students' background knowledge**
- **Use activities that actively engage students and provoke curiosity, desire to know more**
- **Organize students in learning communities**
- **Provide lecture in response to student engagement, inquiry**
- **Use multiple representations (visual models) to support conceptual knowledge**
- **Promote meaningful student discourse**
- **Provide students feedback through intentional formative assessment**

Higher Grades, Less Failure



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Implications of Active Learning Research

"The impact of these data should be like the *Surgeon General's report on Smoking & Health* in 1964, they should put to rest any debate about whether active learning is more effective than lecturing."

Traditional Approach

Let’s learn about parabolas…

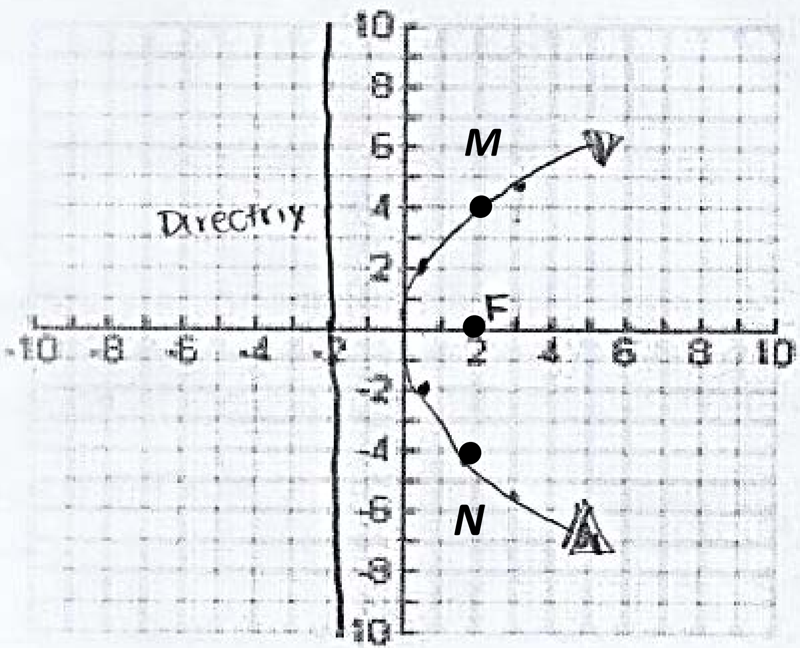
- A *parabola* is a set of points in the plane that are equidistant from a given line, called the *directrix* and a given point not on the directrix, called the *focus*.
- The point of the parabola lying halfway between the focus and the directrix is called the *vertex*. The number p will denote the distance from the vertex to the focus (or directrix).
- If a line be drawn through the focus and parallel to the directrix, it will intersect the parabola at two points that lie a distance 2p from the focus. The line segment connecting these two points is called the *focal chord*. The length of the focal chord is 4p.

Supporting Active Learning in HS Math

- Can use Gizmos Parabolas Activity B
  http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=146

Active Learning Approach

Draw the graph of the parabola (in #6) including the directrix, vertex and focus. Also, draw a line through the focus F that is perpendicular to the axis of symmetry.. The intersection of the line and the parabola let's call M and N.



Find the relationship between D(M,N) and the distance from F to the directrix. Explain in words using math vocabulary.

[Geogebra example]

Reflect

- What are some of the “practices” of mathematics students were engaged in during this activity?
- What was the teacher’s role? What evidence did you see for any of the strategies below?

Mathematical Practices

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Instructional Strategies

Strategy 1: Set Norms and Pose Tasks that Elicit Student Reasoning and Promote Sense-Making

Strategy 2: Provide Information, Guidance & Feedback that Values Specific Student-led Processes and Invites Further Learning

Strategy 3: Recognize and Reduce/Remove Barriers to Learning Mathematics

Taking Action

- What are the implications for our work as teachers of mathematics?

Thank you!!

Questions?

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Additional Resources

Active Learning

- MAA blog series about active learning: http://blogs.ams.org/matheducation/2015/09/10/active-learning-in-mathematics-part-i-the-challenge-of-defining-active-learning/
- http://edtechdev.wordpress.com/2014/06/03/calculus/
- http://www.crlt.umich.edu/sites/default/files/resource_files/Active%20Learning%20Continuum.pdf
- http://web.mit.edu/edtech/casestudies/teal.html
- http://www.educause.edu/sites/default/files/library/presentations/E12/SEM07P/2-Strategies%2BApplied%2Bto%2Bthe%2BALCs.pdf
- http://www2.phy.ilstu.edu/pte/311content/activelearning/activelearning.html
- http://www.jiblm.org/guides/index.php?category=jiblmjournal
- http://blogs.scientificamerican.com/budding-scientist/stop-lecturing-me-in-college-science/

Technology Ideas for Teachers of Mathematics

- http://popplet.com/app/#/83873
- http://www.geogebra.org [Geogebra]
- http://www.shodor.org/interactivate [Shodor Math Interactive Applets]
- http://www.nlvm.org [Nat'l Library of Virtual Manipulatives]
- http://www.wolframalpha.com/widgets/ [Wolfram Alpha widgets]

References

Freeman, S., et al. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Science, 111*(23), 8410-8415. doi :10.1073/pnas.1319030111.

National Council of Teachers of Mathematics (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA.